Quantum superresolution imaging

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KOŁO NAUKOWE OPTYKI I FOTONIKI

LPK Quantum Memories Laboratory

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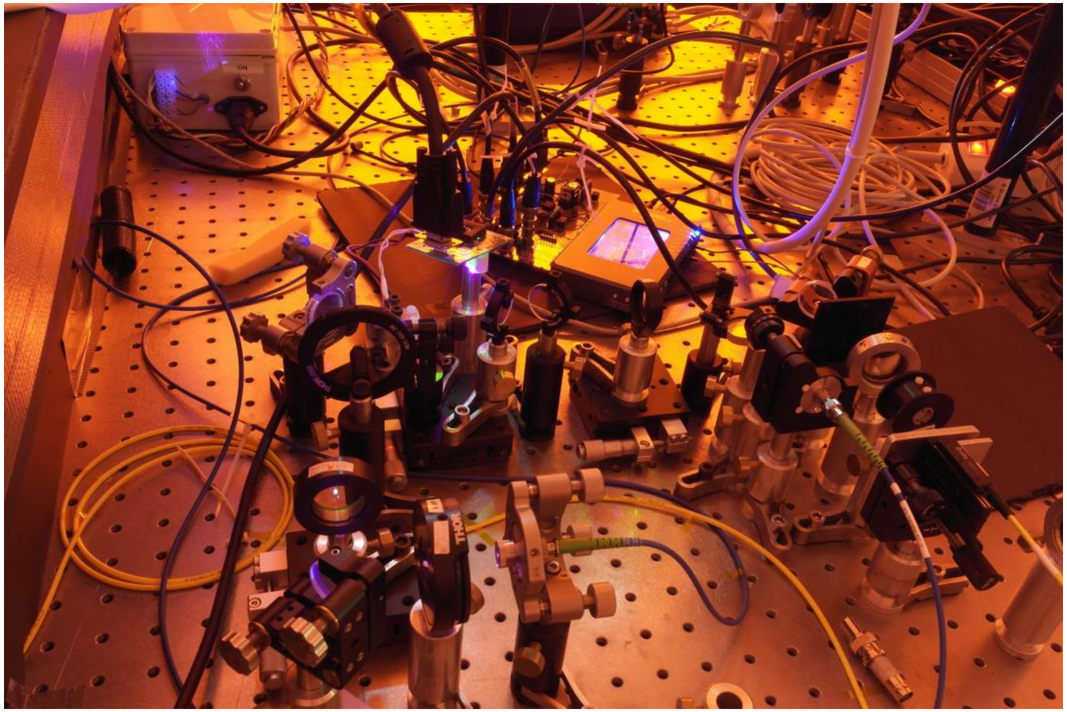

OPTO 2018, Gdańsk

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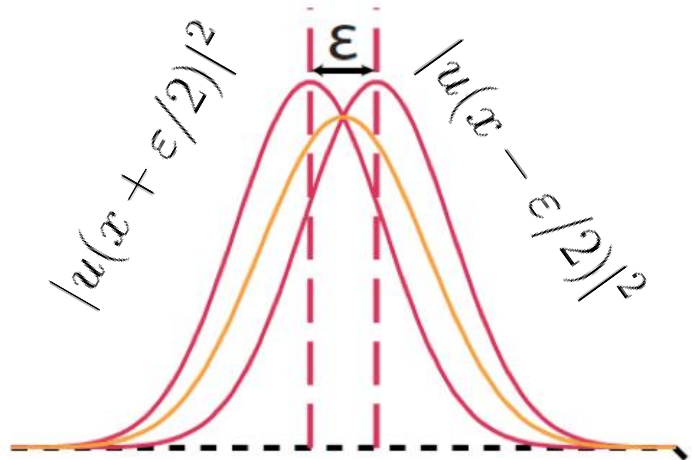
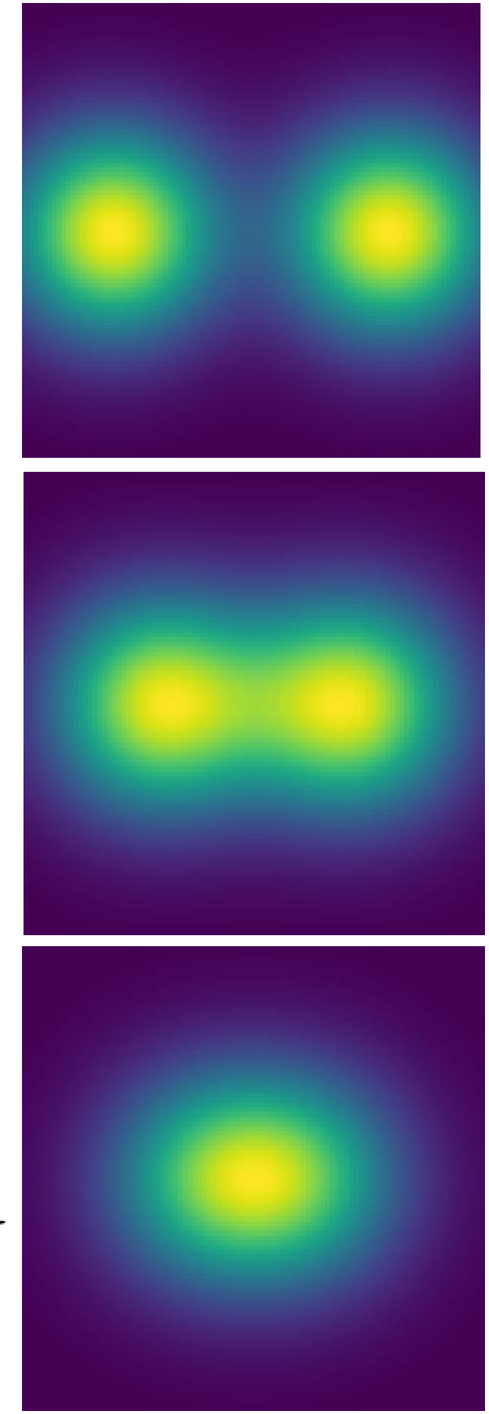
Rayleigh limit

Two point sources:

$$|u(x+\varepsilon/2)|^2 + |u(x-\varepsilon/2)|^2$$





Precision/Uncertainty of estimating source separation by direct imaging:

$$\Delta\varepsilon \propto \frac{1}{\varepsilon\sqrt{N}}$$

$$(\Delta^2\varepsilon)^{-1} \propto \varepsilon^2 N$$

N – number of detected photons

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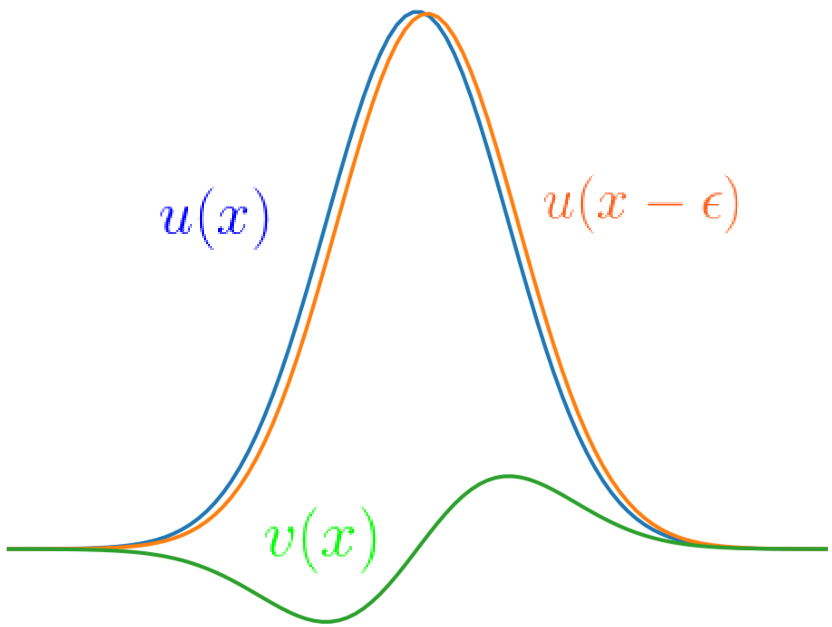
Superresolution techniques

- Maniuplate sample– STED
  - Stimulate emission to increase spatial selectivity
  - Sample heating/degradation
- Use inherent properties of the sample – STORM
  - Use blinkning of the emitters to localize a few in a large sample
  - Not always feasible (e.g. astronomy)

Spatial demultiplexing

$$u(x-\varepsilon) = u(x) + \varepsilon v(x)$$



Idea:

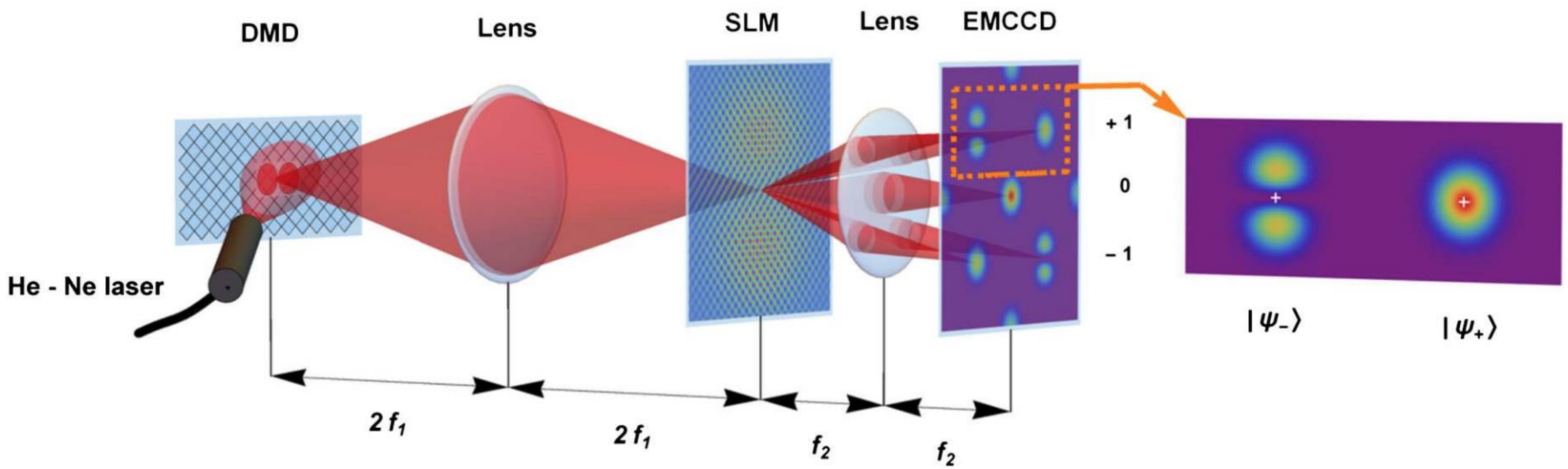
- sort photons into modes *u(x)* and *v(x)*
- Measure the number of photons collected in each mode

Uncertainty: $\Delta\varepsilon \propto \frac{1}{\sqrt{N}}$

Precision: $(\Delta^2\varepsilon)^{-1} \propto N$

M. Tsang, R. Nair, and X.-M. Lu, Phys. Rev. X **6**, 031033 (2016)

Experimental SPADE

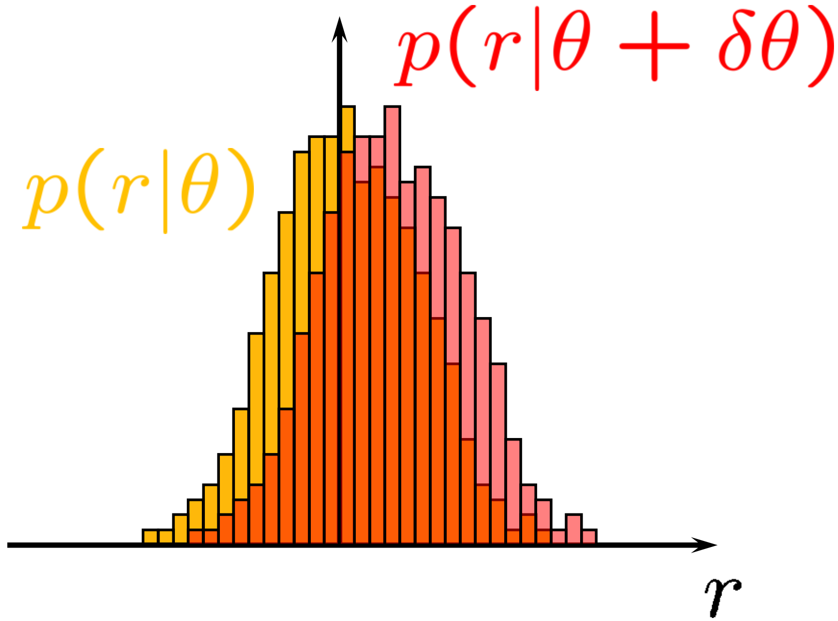


The main problem: how to select the measurement basis?

Paur et al., Optica 3(10), 1144 (2016)

Fisher information

$$F(\theta) = \sum_r p(r|\theta) \left(\frac{\partial}{\partial \theta} \log p(r|\theta) \right)^2$$



Cramér-Rao bound:
for unbiased estimators

$$\Delta\theta \geq \frac{1}{\sqrt{N F(\theta)}}$$

$$(\Delta^2\theta)^{-1} \leq F(\theta) N$$

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Mulitparameter Fisher matrix

$$F_{ij} = \int_{-\infty}^{\infty} \mathrm{d}x \frac{\partial_{\theta_i} p_{\boldsymbol{\theta}}(x) \partial_{\theta_j} p_{\boldsymbol{\theta}}(x)}{p_{\boldsymbol{\theta}}(x)}$$

Cramér-Rao bound:

bound on covariance matrix

(diagonal elements or operator norm)

$$\mathrm{Cov}\boldsymbol{\theta} \geq N \mathbf{F}^{-1}$$

$$\boldsymbol{\theta} = (x_0, \varepsilon)$$

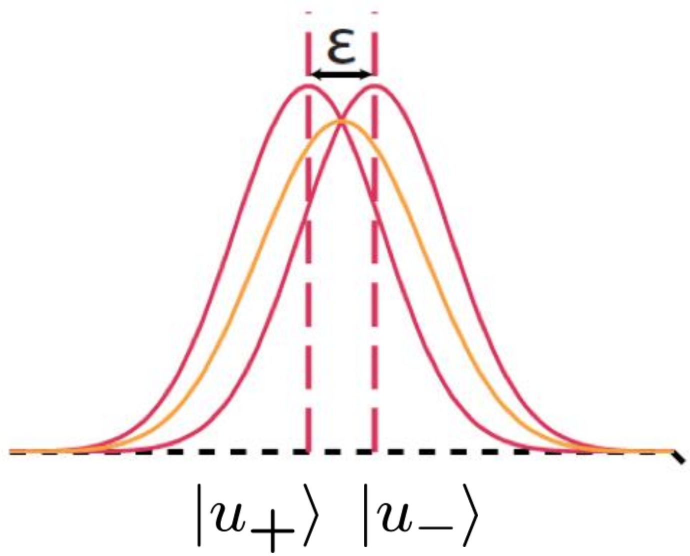
Centroid

Separation

Direct imaging

Quantum approach

Probe state density matrix: $\hat{\varrho} = \frac{1}{2}(|u_+\rangle\langle u_+| + |u_-\rangle\langle u_-|)$



Precision (per photon):

$$(\Delta^2 x_0)^{-1}_{\mathrm{DI}} = 1 - \frac{\varepsilon^2}{4},$$

$$(\Delta^2 \varepsilon)^{-1}_{\mathrm{DI}} = \frac{\varepsilon^2}{8},$$

Dramatic fall of precision of seperation estimation with separation

Rayleigh curse

Ultimate bound

Quantum Cramer-Rao bound – optimized over all possible states and measurements

Precision = Inverse Uncertainties^2 per photons

$$\mathrm{Cov}\boldsymbol{\theta} \geq N\mathbf{F_Q}^{-1}$$

$$(\Delta^2 x_0)_{\mathrm{Q}}^{-1} = 1 - \frac{\varepsilon^2}{4},$$

$$(\Delta^2 \varepsilon)_{\mathrm{Q}}^{-1} = \frac{1}{4},$$

Constant precision of separation estimation – much more information available

Better scheme needed!

Rehacek et al., Phys. Rev. A 96, 062107 (2017)

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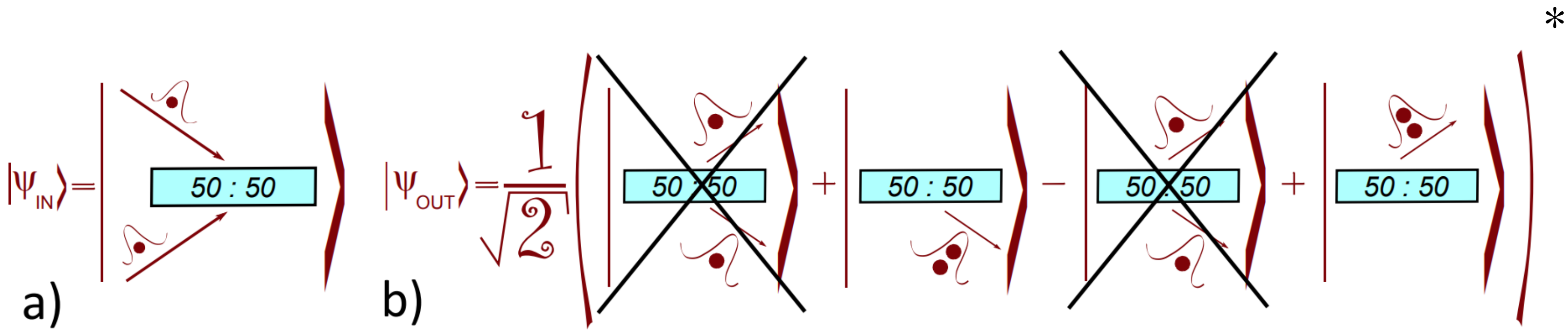
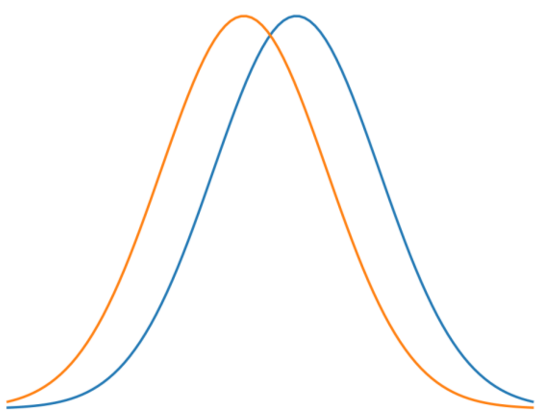
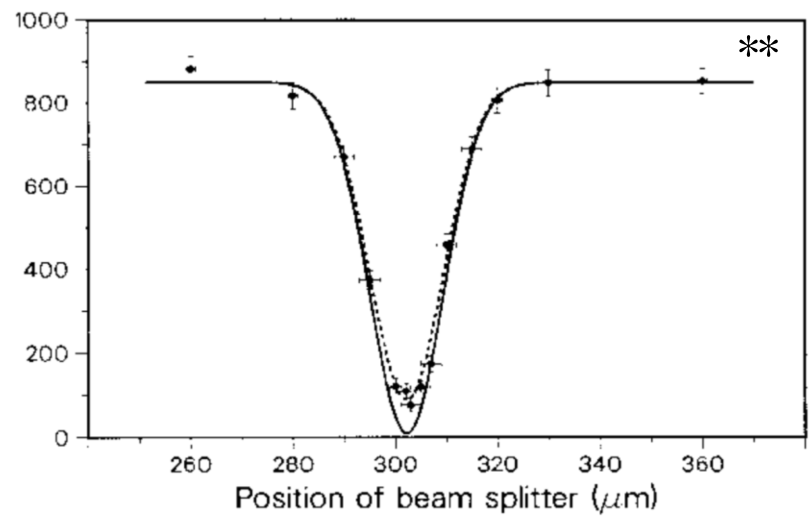
Hong-Ou-Mandel interference



Nonidentical photons: these two won't cancel out!

Example: spatial distinguishability

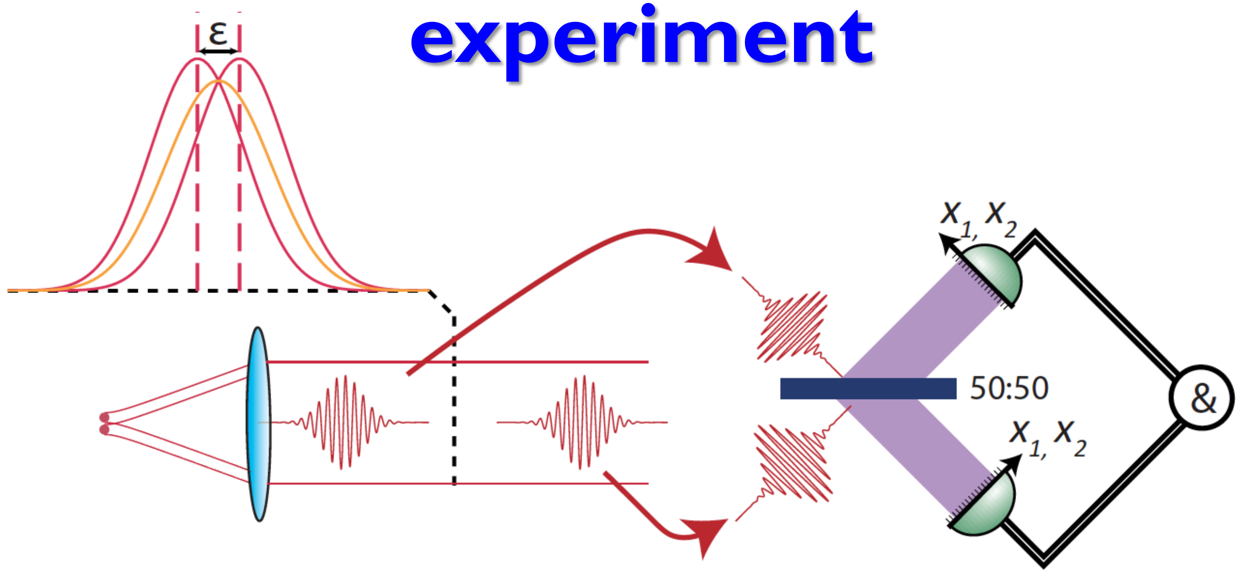


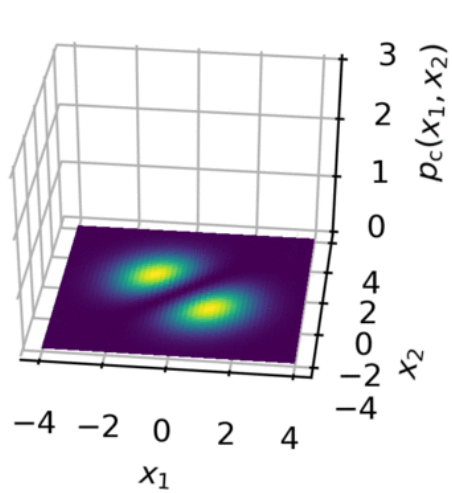


Number of coincidences proportional to 1 - Overlap

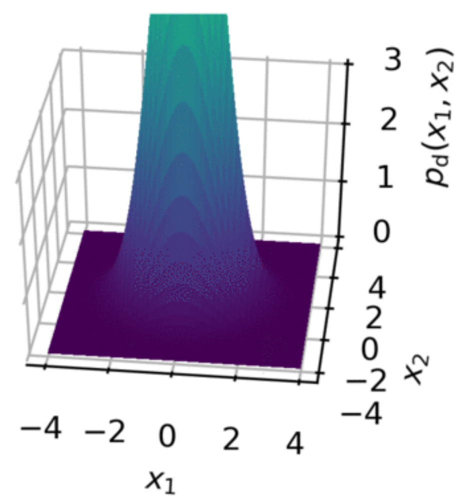
*M. Jachura, *Nieklasyczna interferencja par fotonów generowanych w falowodzie nieliniowym*
** Hong, Ou & Mandel, *PRL* 59, 2044 (1987)

The Quantum Superresoltuion experiment





Cross-coincidences



Double events

MP, S. Borówka, K. Boroszko, W. Wasilewski, K. Banaszek, R. Demkowicz-Dobrzański,
arXiv:1803.07096

Two-photon scheme

Probe state: $\rho^{\otimes 2}$

$$\hat{\varrho} = \frac{1}{2}\left(|u_+\rangle\langle u_+| + |u_-\rangle\langle u_-|\right)$$

$$(\Delta^2 x_0)^{-1}_{2\mathrm{P}} = 1 - \frac{\varepsilon^2}{4},$$

$$(\Delta^2 \varepsilon)^{-1}_{2\mathrm{P}} = \begin{cases} \frac{1}{8} + \frac{5}{128}\varepsilon^2 & \mathcal{V} = 1 \\ \frac{4-\mathcal{V}^2}{32(1-\mathcal{V}^2)}\varepsilon^2 & \mathcal{V} < 1 \end{cases},$$

- Lower than in qCRB, but ε-independed separation estimation precision for V=1
- Constant factor enhancement for finite visibility V
- Unaffected centroid estimation precision (with respect to DI)

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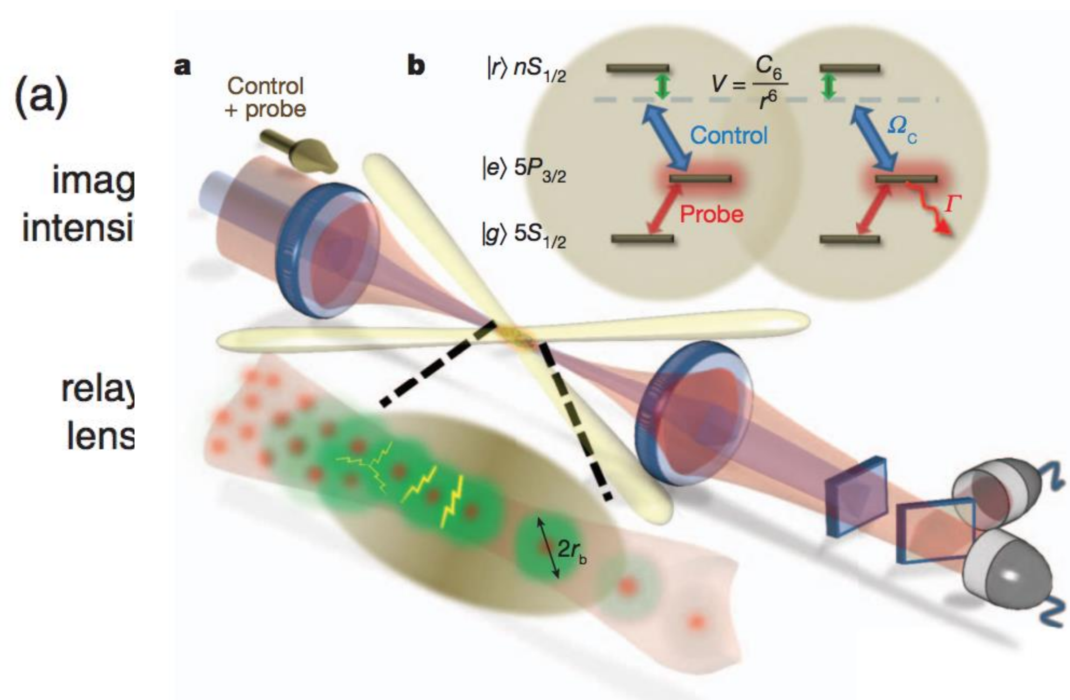
Towards experimental realization

- Spatially-resolved single-photon detection ✓
- Multimode quantum memory ✓
- Photon-number QND/photon-number selective storage ✓
- Spatial-mode preserving photon-number QND *In progress…*



T. Peyronel *et al.*, Nature **488**, 57 (2012)

M. Parniak *et al.*, Nat. Commun. **8**, 2140 (2017)

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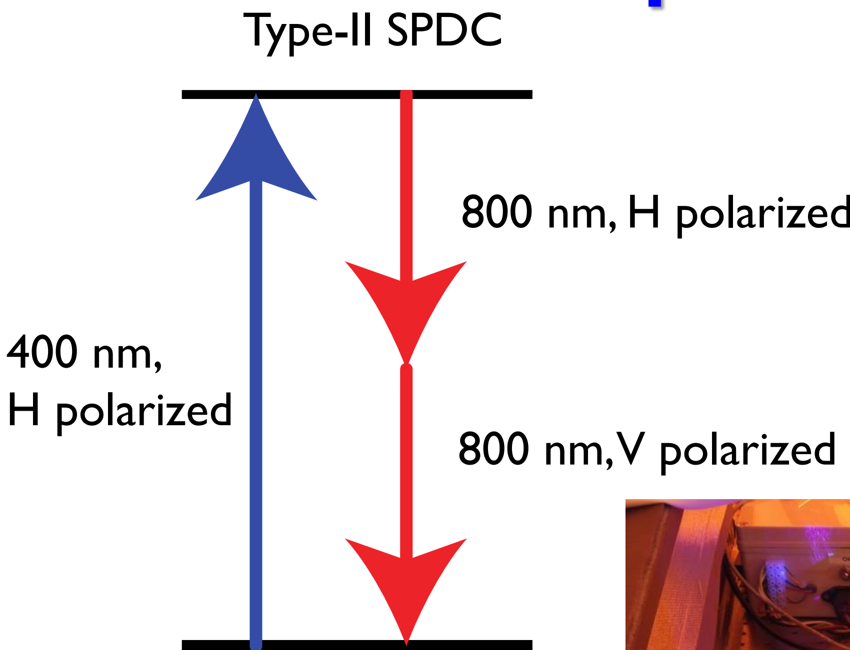
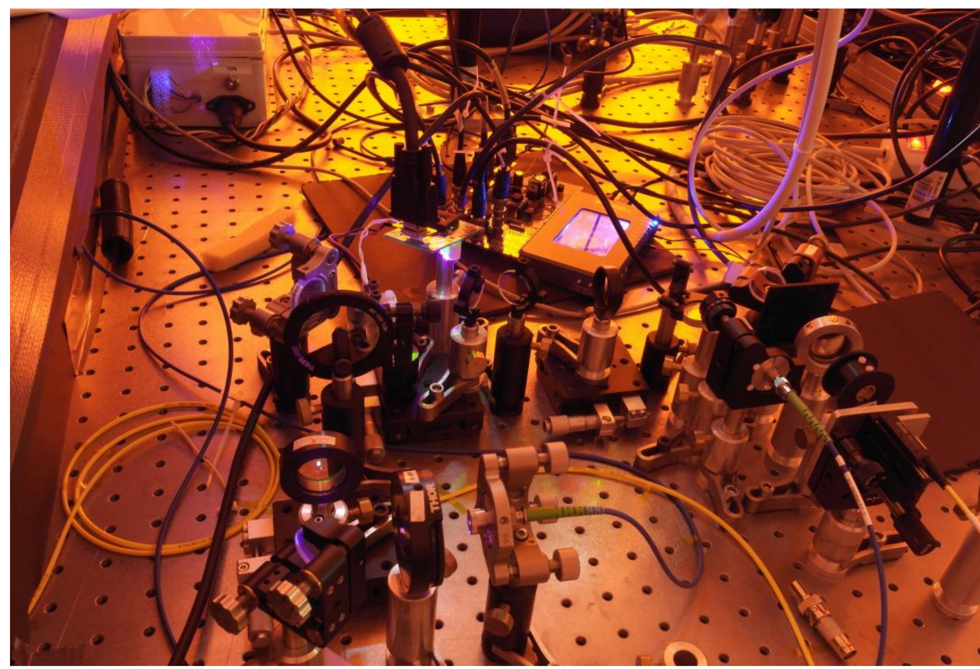
How to make a pair of indistinguishable photons?





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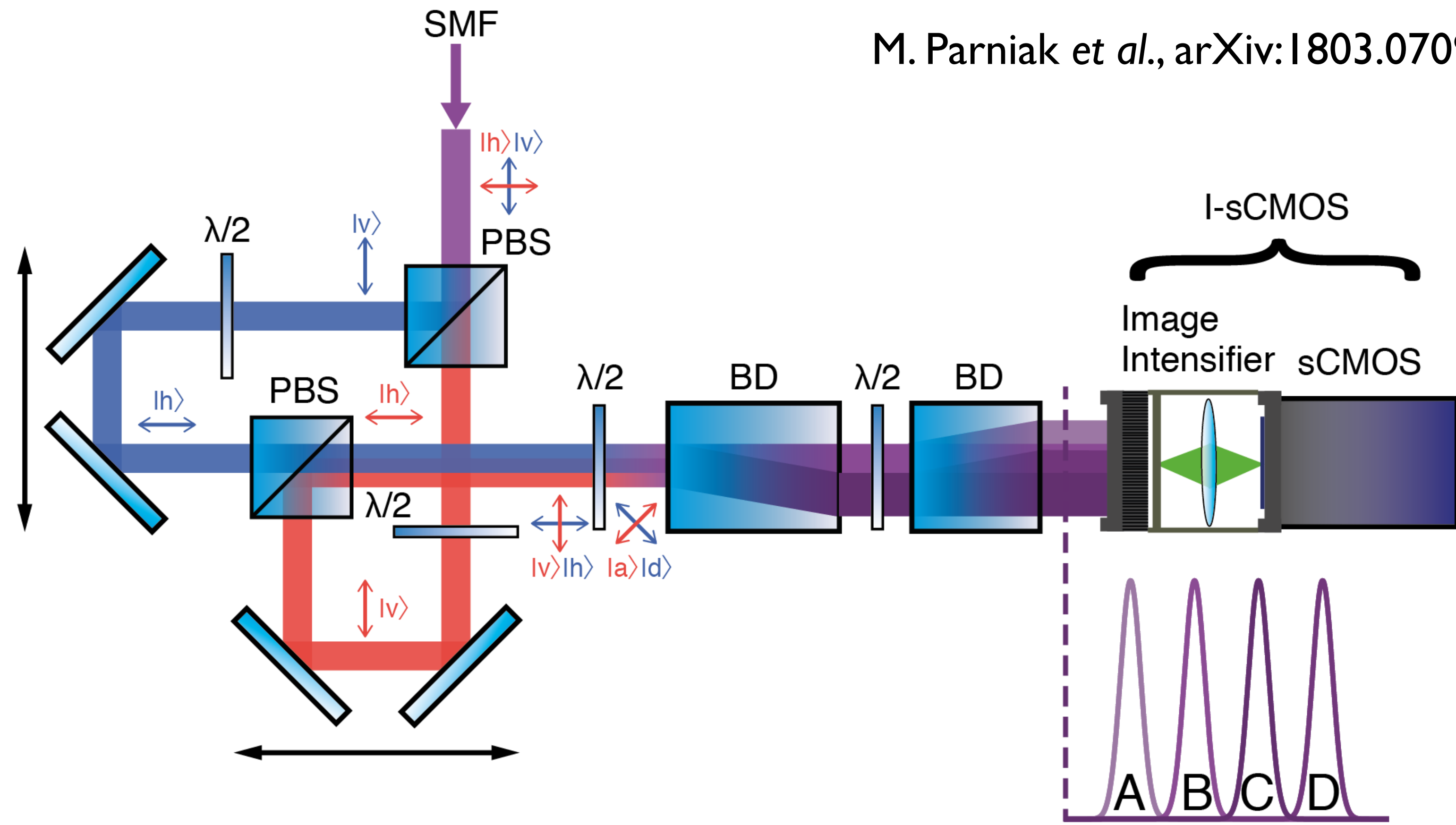
Proof-of-princple experiment

M. Parniak *et al.*, arXiv:1803.07096



- Generates probe states with variable separation from indistinguishable photon pairs
- Ensures constant spectro-temporal visibility of HOM interference thanks to invariant arm lengths
- Detects coincidences and double events using four distinct camera regions

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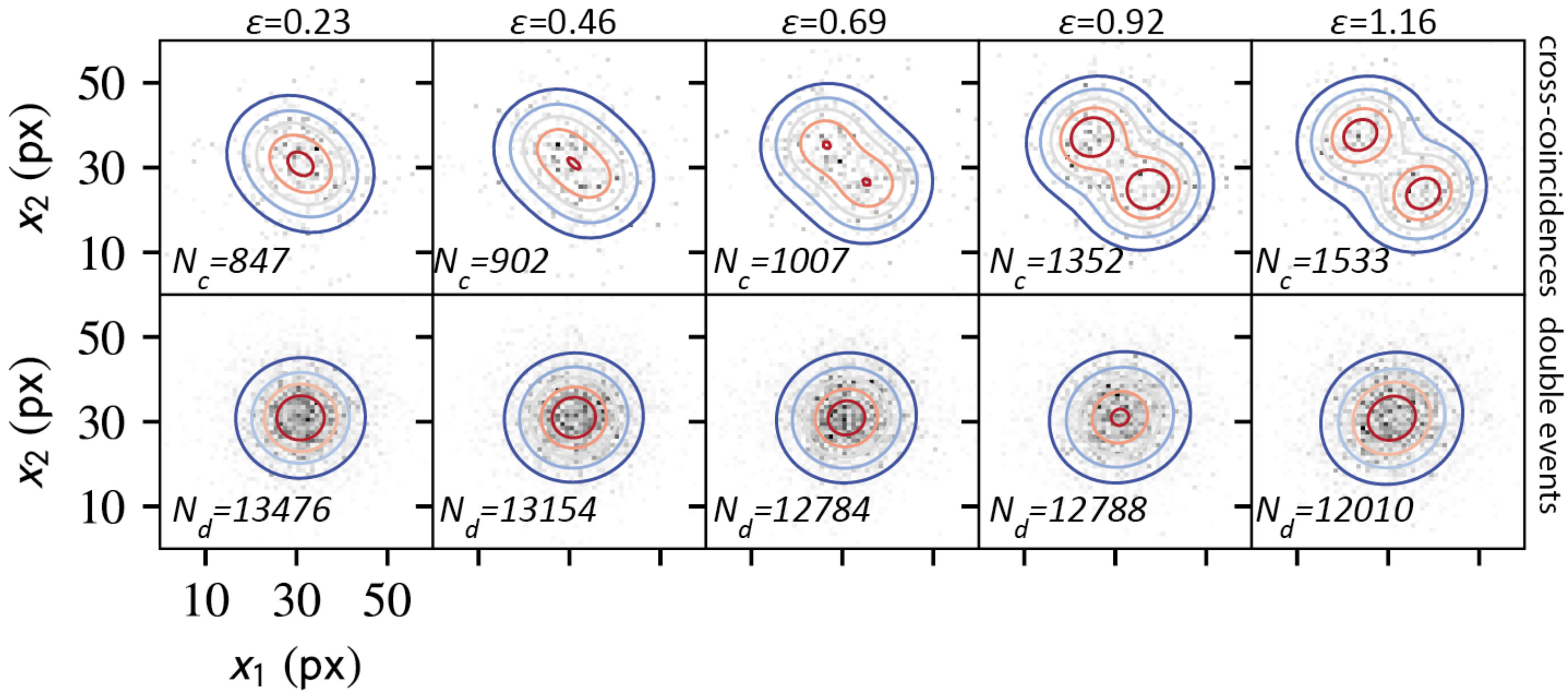
Spatially-resolved coincidences



- Separation and centroid are estimated both from shape and coincidences to doubles ratio
- Maximum likelihood estimator (which saturates the Cramer-Rao bound for large N) is used to retrieve the best fitting parameter

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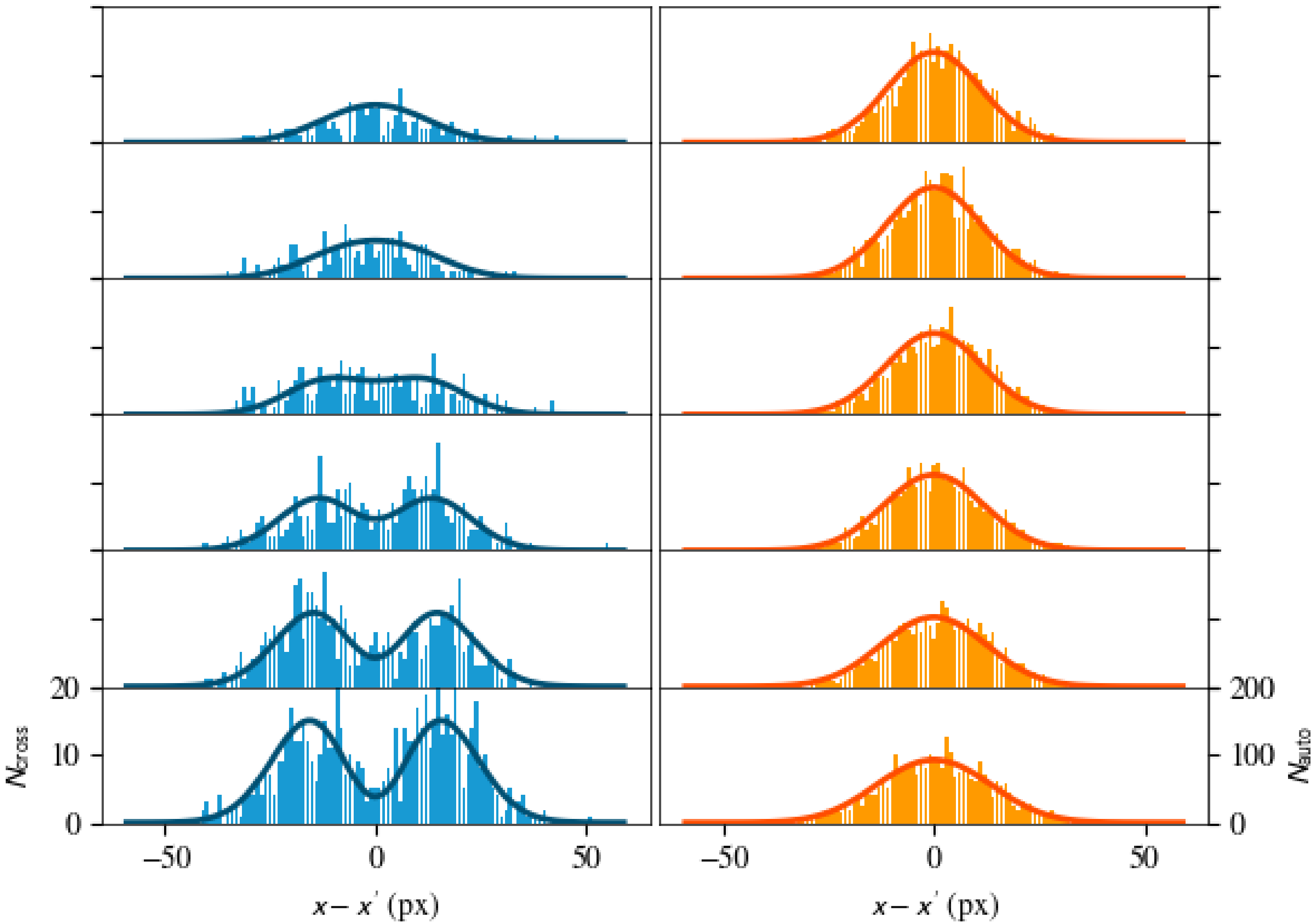
Spatially-resolved coincidences



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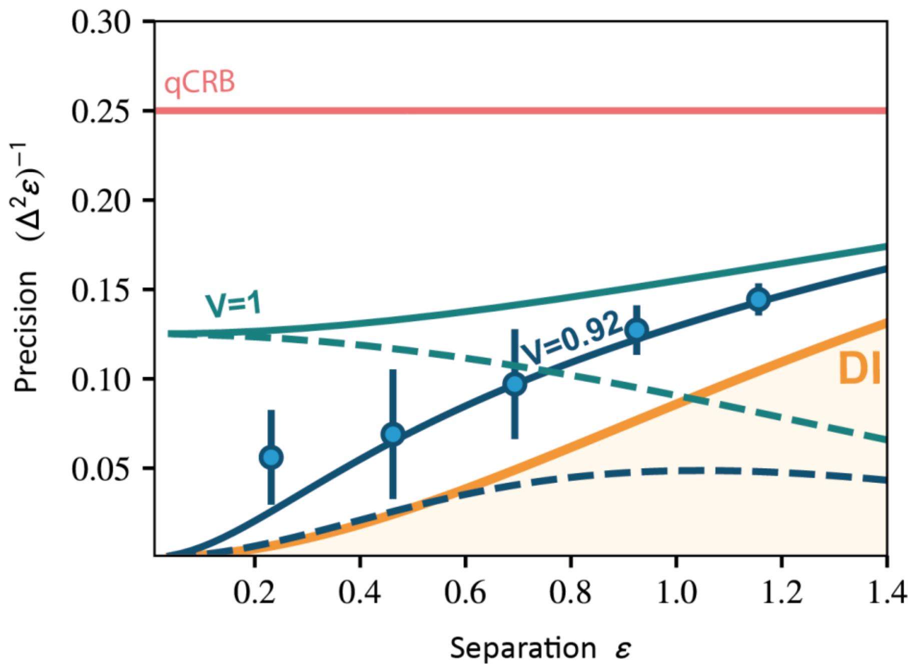
Separation estimation



with camera

no spatial information

Direct Imaging

Perfect (V=1) two-photon scheme

Ultimate (quantum Cramer-Rao) precision bound

Realistic two-photon scheme

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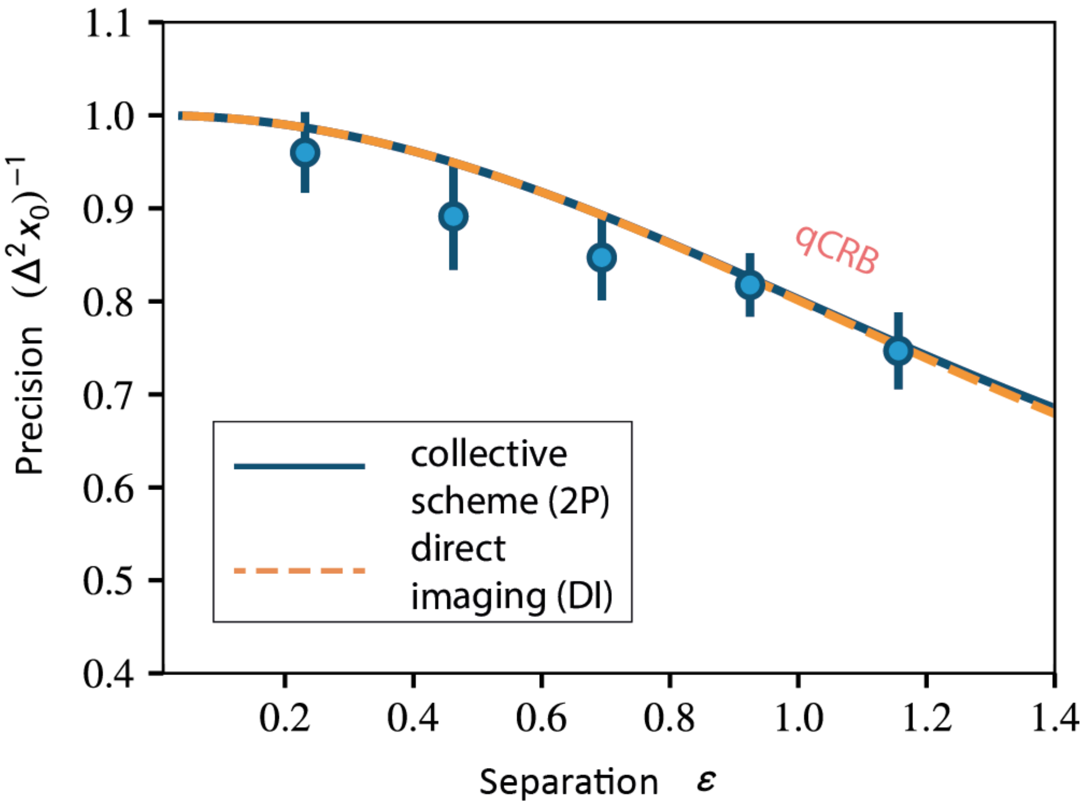
Centroid estimation



Precision of centroid estimation fully preserved

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    - Quantum information theory
    - Electronics, optoelectronics

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*** This work**