Three-mode interaction in Raman atom-light interface

Michał Parniak[1]*, Daniel Pęcak[2] and Wojciech Wasilewski[1]

[1]Institute of Experimental Physics, Faculty of Physics, University of Warsaw
[2]Institute of Physics, Polish Academy of Sciences
*michal.parniak@fuw.edu.pl

Abstract

We theoretically analyze the properties of a **Raman quantum light-atom interface** in long atomic ensemble and its applications as a **quantum memory** or **two-mode squeezed state generator**. We consider the weak-coupling regime and include both Stokes and anti-Stokes scattering and the effects of **Doppler broadening** in buffer gas assuming **frequent velocity-changing collisions**. The model we present bridges the gap between the Stokes only and anti-Stokes only interactions providing simple, universal description in a **temporally and longitudinally multimode** situation.

Introduction

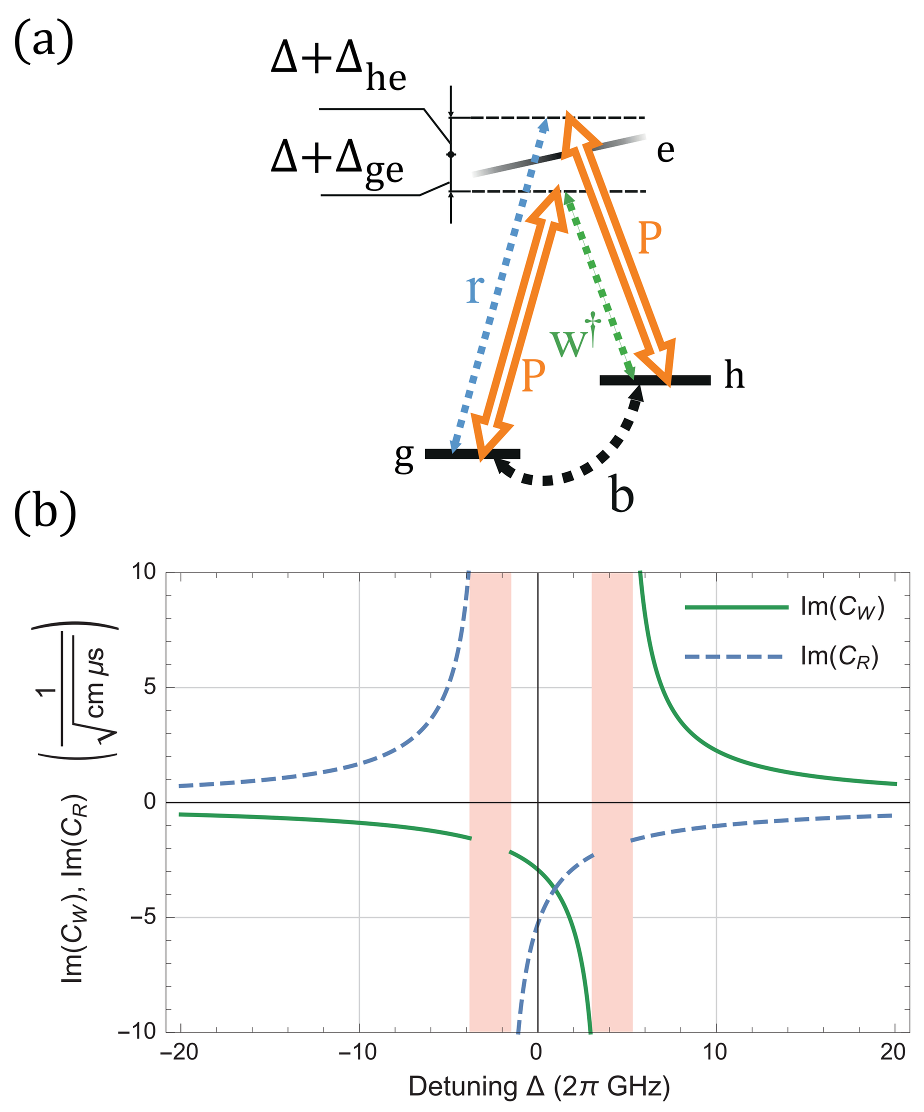

Figure 1: (a) Strong linearly polarized pump of amplitude P couples two separate hyperfine components of the ground state, $|g\rangle$ and $|h\rangle$ via the excited state $|e\rangle$. Pump is detuned from the $|g\rangle-|e\rangle$ transition by $\Delta+\Delta_{ge}$ and from the $|h\rangle-|e\rangle$ transition by $\Delta+\Delta_{he}$, where $\Delta=0$ means the pump is tuned to the line centroid. (b) Coherent scattering coefficients $c_R(\Delta)$, $c_W(\Delta)$ for rubidium 87 D1 line ($\lambda\approx 795$ nm). We take two hyperfine components of the ground state as levels $|g\rangle=|F=1,m_F=0\rangle$ and $|h\rangle=|F=2,m_F=0\rangle$. We take the atom number density equal $N=10^{12}$ cm^{-3} and pump field Rabi frequency equal to the natural linewidth $\Gamma/2\pi=5.75$ MHz.

Off-resonant Raman interaction is a vividly developing approach to quantum memory. Two basic modes of operation can be distinguished:

- Photons are created in an external source are stored in memory via anti-Stokes scattering (read-in).
- Pair of photons and atomic excitations are created in the memory via Stokes scattering

Stored atomic coherence that takes on the form of spinwave is created and can be converted back to light in the read-out process. The spinwaves can be stored and then further manipulated. In realistic experimental situations however, both Stokes and anti-Stokes scattering are present. If one process is required, the other will always maliciously prevail. We analyze the interaction of both Stokes and anti-Stokes sidebands with atomic excitations.

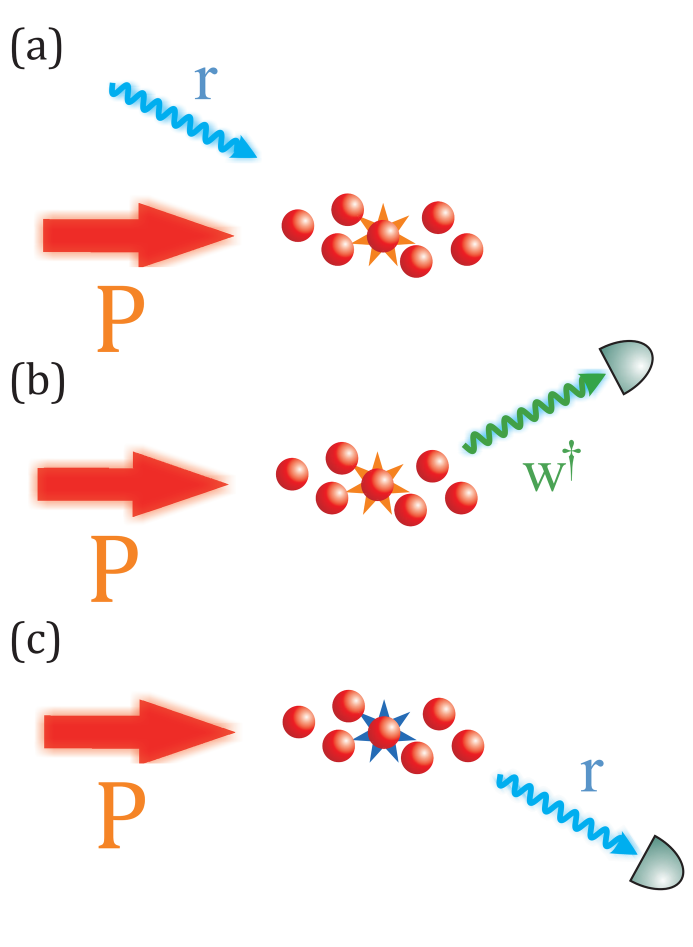

Figure 2: Typical experimental situations include (a) generating photons externally and imprinting them onto quantum memory in the read-in process, (b) generating pair of photons and atomic excitations and (c) read-out of stored atomic excitations.

Evolution of Fields

We consider a full of interaction, including collision, as together with warm atomic ensembles an inert buffer gas, such as neon, krypton or xenon is used to make the motion of atoms diffusive and consequently prolong the lifetime of stored spatial mode [1, 2]. Collisions transfer atoms from velocity class v' into v with probability per unit time given by $\gamma_v K(v\leftarrow v')\mathrm{d}v'$, where γ_v is the collision rate and $K(v\leftarrow v')$ is the collisional kernel [3].
Adiabatic elimination lead to a set of Maxwell-Bloch equations [4] describing the interaction. They can be cast in terms of field operators in a reference frame co-moving with weak light [5]:

$$\frac{\partial\hat{r}(z,t)}{\partial z}=\int\sqrt{g(v)}c_R(\Delta+kv)\hat{b}(z,t,v)\mathrm{d}v \tag{1}$$

$$\frac{\partial\hat{w}^\dagger(z,t)}{\partial z}=\int\sqrt{g(v)}c_W(\Delta+kv)\hat{b}(z,t,v)\mathrm{d}v \tag{2}$$

$$\begin{aligned}\frac{\partial\hat{b}(z,t,v)}{\partial t}&=\sqrt{g(v)}c_W^*(\Delta+kv)\hat{w}^\dagger(z,t)\\+\sqrt{g(v)}c_R(\Delta+kv)\hat{r}(z,t)&-s(\Delta+kv)\hat{b}(z,t,v)\\+\gamma_v\int K(v\leftarrow v')&\sqrt{\frac{g(v')}{g(v)}}\hat{b}(z,t,v')\mathrm{d}v',\end{aligned} \tag{3}$$

where $g(v)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{v^2}{2\sigma^2})$. If we assume that the collisions are fast compared to the Raman interaction $\gamma_v\gg Lc^2_{R,W}$, then the velocity dependence of number operator of atoms in $|h\rangle$ state represented by $\hat{b}^\dagger(z,t,v)\hat{b}(z,t,v)$ remains close to thermal equilibrium. Consequently, we may separate out the known, Gaussian velocity dependence and assume $\hat{b}(z,t,v)=\hat{b}(z,t)\sqrt{g(v)}$.
The equation (3) for $\hat{b}(z,t,v)$ can now be integrated formally and averaged over velocity distribution $\hat{b}(z,t)=\int\sqrt{g(v)}\hat{b}(z,t,v)\mathrm{d}v$, yielding:

$$\hat{b}(z,t)=\hat{b}(z,0)+\int_0^t e^{-(t-t')\bar{s}}\left(\bar{c}_R\hat{r}(z,t')+\bar{c}_W^*\hat{w}^\dagger(z,t')\right)\mathrm{d}t', \tag{4}$$

This result corresponds to taking into account only the fundamental velocity mode [6]. Substituting the result to Eqs. (1)–(2) gives the following equation of evolution for the photonic modes:

$$\frac{\partial}{\partial z}\begin{pmatrix}\hat{r}(z,t)\\\hat{w}^\dagger(z,t)\end{pmatrix}=\begin{pmatrix}\bar{c}_R\\\bar{c}_W\end{pmatrix}\hat{b}(z,0)e^{-\bar{s}t}+\int_0^t e^{-(t-t')\bar{s}}\begin{pmatrix}\bar{c}_R^2 & \bar{c}_R\bar{c}_W^*\\\bar{c}_W\bar{c}_R & |\bar{c}_W|^2\end{pmatrix}\begin{pmatrix}\hat{r}(z,t')\\\hat{w}^\dagger(z,t')\end{pmatrix}\mathrm{d}t'. \tag{5}$$

The solution of equations (4)–(5) takes on a form of a linear transformation between the input and output quantum fields.

Decomposition

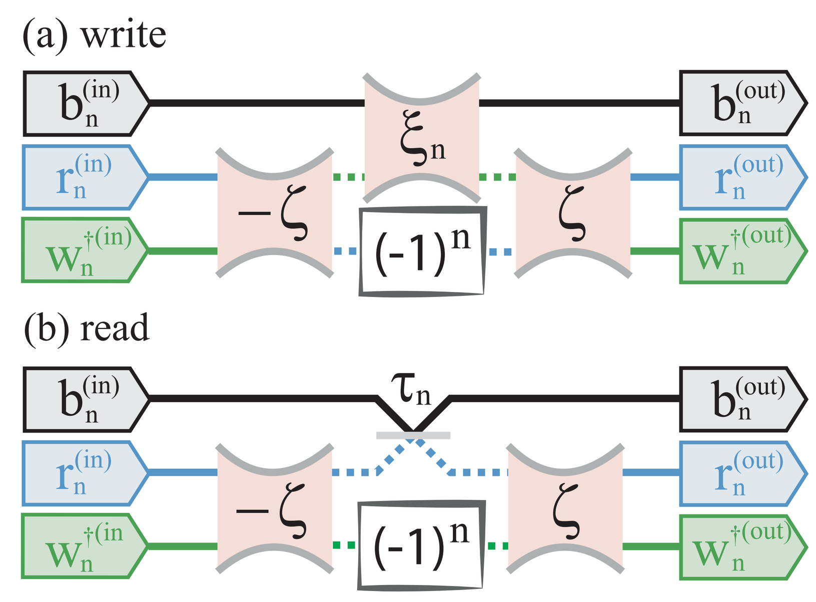

Figure 3: Diagrammatic representation of Raman interaction for (a) $\kappa>0$ and (b) $\kappa<0$. Squeezing $\zeta=\operatorname{atanh}|\frac{\bar{c}_R}{\bar{c}_W}|$ for $\kappa>0$ and $\zeta=\operatorname{atanh}|\frac{\bar{c}_W}{\bar{c}_R}|$ for $\kappa<0$.

Hyperbolic rotation $R(\zeta)$ between the photonic modes is used to diagonalize the matrix from Eq. (5). Modes $\hat{c}^\dagger(z,t)$ and $\hat{d}(z,t)$ are defined as linear combinations of $\hat{r}(z,t)$ and $\hat{w}^\dagger(z,t)$. The mode $\hat{d}(z,t)$ turns out to be decoupled: $\hat{d}(L,t)=\hat{d}(0,t)$. The equations coupling atoms $\hat{b}(z,t)$ with photonic mode $\hat{c}(z,t)$ are the same as in single-sideband Raman scattering [4, 7, 5] and their solution reads:

$$\hat{b}(z,T)=\int_0^T\hat{c}^\dagger(0,t')\sqrt{\kappa}\Sigma_0(z,T-t')\mathrm{d}t'+\int_0^z\hat{b}(z',0)[e^{-Ts}\delta(z-z')+T\Sigma_1(z-z',T)]\mathrm{d}z' \tag{6}$$

$$\hat{c}^\dagger(L,t)=\int_0^L\hat{b}(z',0)\sqrt{\kappa}\Sigma_0(L-z',t)\mathrm{d}z'+\int_0^t\hat{c}^\dagger(0,t')[L\Sigma_1(L,t-t')+\delta(t-t')]\mathrm{d}t' \tag{7}$$

where the interaction strength is measured by $\kappa=\bar{c}_R^2+|\bar{c}_W|^2$ and $\Sigma_1(z,t)=\sqrt{\frac{\kappa}{zt}}I_1(2\sqrt{\kappa zt})$, $\Sigma_0(z,t)=I_0(2\sqrt{\kappa zt})$.
The mode basis for atomic and photonic fields and squeezing ξ_n or beamsplitter transmission τ_n for the central operation are calculated from SVD of Green functions [5, 8, 9]. For weak interaction we calculate squeezing $\xi_n=(\kappa LT)^{n+1/2}$ if $|\bar{c}_W|>|\bar{c}_R|$ or beamsplitter transmission $\tau_n=(-\kappa LT)^{n+1/2}$ if $|\bar{c}_W|<|\bar{c}_R|$. The mode functions are given by Legendre polynomials (see figure below).

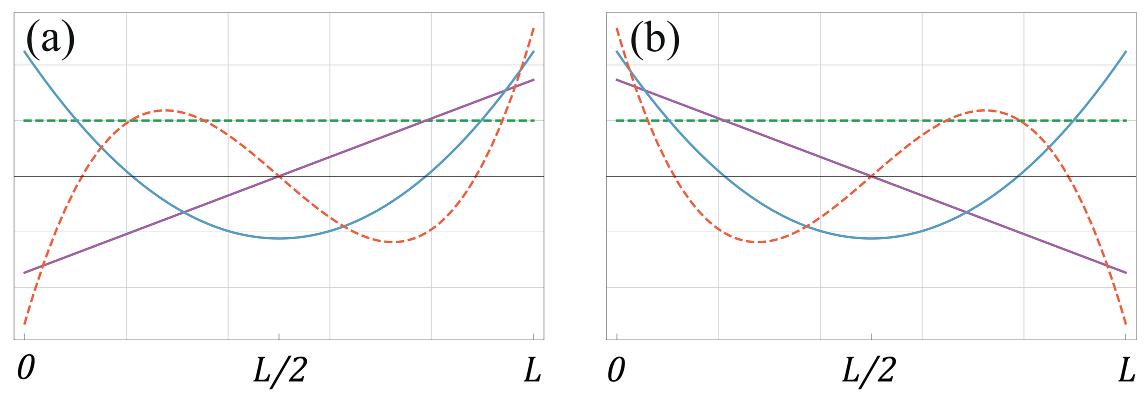

Figure 4: First four (a) input and (b) output atomic modes.

Reduction

In the regime of very weak coupling, we can find triples of modes that interact only with each other, as input and output modes have the same shape

Universal picture

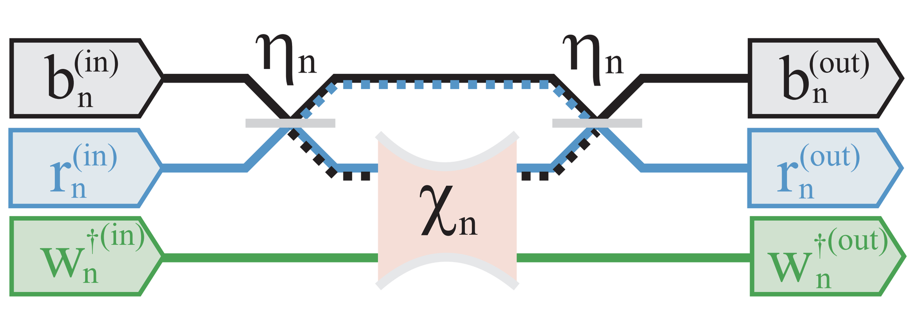

Figure 5: Universal representation of Raman interaction.

We can find a single representation for both cases from Fig. 3. The parameters of interaction for fundamental mode are:

$$\cosh(\chi)=\begin{cases}\cosh^2(\zeta)\cosh(\xi)-\sinh^2(\zeta) & \text{for } |\bar{c}_W|>|\bar{c}_R|\\\cosh^2(\zeta)-\sqrt{1-\tau^2}\sinh^2(\zeta) & \text{for } |\bar{c}_W|<|\bar{c}_R|\end{cases} \tag{8}$$

$$\eta=\begin{cases}\sqrt{\frac{1+\cosh(\xi)}{1+\cosh(\chi)}} & \text{for } |\bar{c}_W|>|\bar{c}_R|\\\sqrt{\frac{1+\sqrt{1-\tau^2}}{1+\cosh(\chi)}} & \text{for } |\bar{c}_W|<|\bar{c}_R|\end{cases} \tag{9}$$

This picture allows us to easily find output state of fields in general form:

$$\mathcal{U}|000\rangle_{\mathrm{in}}=\frac{1}{\cosh(\chi)}\sum_{j=0}^{\infty}\tanh^j(\chi)\times\sum_{k=0}^{j}\sqrt{\binom{j}{k}}(1-\eta^2)^{k/2}\eta^{j-k}|j-k,k,j\rangle_{\mathrm{out}} \tag{10}$$

References

[1] M. Parniak and W. Wasilewski, Appl. Phys. B **116**, 415 (2014).
[2] R. Chrapkiewicz, W. Wasilewski, and C. Radzewicz, Opt. Commun. **317**, 1 (2014).
[3] B. H. McGuyer, R. Marsland III, B. A. Olsen, and W. Happer, Phys. Rev. Lett. **108**, 183202 (2012).
[4] M. G. Raymer and J. Mostowski, Phys. Rev. A **24**, 1980 (1981).
[5] M. G. Raymer, J. Mod. Opt. **51**, 1739 (2004).
[6] S. Kryszewski and J. Gondek, Phys. Rev. A **56**, 3923 (1997).
[7] M. G. Raymer, I. A. Walmsley, J. Mostowski, and B. Sobolewska, Phys. Rev. A **32**, 332 (1985).
[8] W. Wasilewski and M. G. Raymer, Phys. Rev. A **73**, 063816 (2006).
[9] J. Kołodyński, J. Chwedeńczuk, and W. Wasilewski, Phys. Rev. A **86**, 013818 (2012).

Acknowledgements

This work was supported by the National Science Centre (Poland) Grant No. DEC-2011/03/D/ST2/01941 and by the Polish Ministry of Science and Higher Education "Diamentowy Grant" Project No. DI2013 011943.

Diamentowy Grant

NATIONAL SCIENCE CENTRE POLAND